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Information flow in networked system with leaderless structure

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ABSTRACT

The research on the consensus protocol design has been carried out for the leaderless networked system in this paper. When a specific node detects new information from outside, protocol for the new action needs to be designed. For the multiple information access, nodes behavior in networked system show as group action. For the nodes inside of group show the similar pattern, but the nodes locate in the middle of different groups face with challenge to decide. In this research, we provide the measure to make inclusion to the specific group for the nodes inbetween groups. In order to discriminate group inclusion for intermediate nodes, similarity measure has been proposed with each node characteristics. To clarify different agent's characteristic, which is their preference on different leader groups, the social weighting concept will be introduced. Depending on the characteristic function, we could decide the grouping of inbetween agents with Shapley value method. The grouping consensus has been realized by the Ofalti-Saber algorithm in both single input and multi-inputs situation for leaderless network, and through the simulation, the algorithm is verified to perform well in the mentioned situations.

1. INTRODUCTION

Recently, research on consensus within network has been emphasized on practical application such as unmanned air vehicle (UAV), rendezvous problem, and social agreement as well [1-5]. In the networked system, each node is denoted by agent, then it represents for human in social network, vehicle or fish in school [3-5]. Hence, multi-agent system analysis has been studied by for the synchronization, group action, and cooperative control area [5-7]. With the help of graph theory, networked system is expressed via nodes and edge.

Research on consensus has been originated from graph theory back ground [14]. And it was applied network consensus, and the analysis is applicable final consensus whenever different information is obtained by other agents.

As the application of the multi-agent research, it has been widened by cooperative teamwork in civilian and military application has caught attention for the autonomous vehicles, specifically in unmanned system. Research necessity comes from the distributed and complex system control and information sharing for the goal. One of application is multi-agent system analysis and design. Its application and executions such as formation control, rendezvous, attitude alignment, flocking, task and role assignment, air traffic control require for the individual vehicles share a consistent view of the objectives and the world. Hence, consensus idea is the one of fundamental background to impose similar dynamic characteristics on the status of agents, and reach to the agreement. For the continuous or communication bandwidth is wide case, information state update of each agent follows differential equation. Otherwise, information state follows difference equation when the communication data is arrived with discrete packets.

In this paper, we will use the Olfati-Saber algorithm to deal with the multi-agent network alignment and grouping consensus. This algorithm could be used to solve both the single informed network flocking consensus and the multi-informed network consensus, which is also a verification of the ILR for multi-input reaction using another

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algorithm [10]. The Olfati-Saber algorithm really converge ideally even for the large number of the agent group as long as the perception radius is large enough and the neighbors of the leader are relatively sufficient [12].
The Shapley value is introduced in this paper as well to deal with the inbetween agents, which originates from the game theory methodology to obtain better performance in the control and network. By calculating the Shapley value for different grouping of the inbetween agents, we could make the network reach a more efficient grouping consensus performance.
The paper is organized as follow. In next section, preliminary on graph theory, networked system, Shapley value and implicit leadership are illustrated. In Section 3, consensus protocol on leaderless networked is introduced, and the calculation of contribution to the group for the agent in the middle of each group has been carried by the simple example. Simulation of 10 agents in the networked system has been done for two external commends. Finally, conclusions are followed in Section 4.

2. PRELIMINARIES

In this section, elementary Graph theory is illustrated to understand the networked system. And the networked model is introduced based on Laplacian matrix. And the cooperative game and Shapley value are illustrated which are effective in multi-agent cooperation. At the same time, the performance of informed agents is described by implicit leadership reinforcement algorithm.

A. Notations: Graph theory

A graph G constitutes with the pair of sets (V, E). V is defined as vertices or nodes, it is also represented as agent in networked system. E is called edge, which represents the connection between nodes. Hence, $V = (v_1, \ldots, v_n)$ and n is the number of nodes. And let $E = [e_{ij}]$, then e_{ij} illustrates edge between node i and j. For all nodes n, edge E can be expressed by incidence matrix, adjacency matrix and Laplacian [5].
For n nodes of the graph G, adjacency matrix constitutes $n \times n$ with e_{ij} edge from node i to j. And it is symmetric if the graph is undirected. Undirected graph diagonal and off diagonal elements are defined as $l_{ii} = \sum_{j=1, j\neq i}^{n} e_{ij}$ and $l_{ij} =- e_{ij}$, for all $i \in N$, respectively. Matrix $[l_{ij}] = L$ is called the Laplacian matrix. And Laplacian matrix includes many information on network system. For example, network connectivity is represented by the second smallest eigenvalue of L [5].
Consensus algorithm can be written in Laplacian matrix form as follow

$$\dot{x} =- Lx. \tag{1}$$

With the Graph theory knowledge, networked system modelling can be derived in the next subsection.

B. Network model

Leader-Follower model has been proposed by many researchers [6 - 8 and the reference there in]. In their results, leader is selected by the consideration of information accessibility or information flow, and also shows that followers reach to the consensus with relevant protocol. Specifically, many results showed that they asymptotically converges with homogeneous structures of protocol [6-8]. Zhang and *et. al* proposed time varying nonlinear model, which described the actual situation with similar [9]. For power grid, time invariant model was also proposed by Zhang and Chow [8- 10]. By the consideration of irrigation system,

$$\text{Follower, } \dot{x}_i = \sum_{l=1}^{n} l_{il}\, x_l \tag{2}$$

$$\text{Leader, } \dot{x}_j = \sum_{l=1}^{n} l_{jl}\, x_l + \varepsilon \sum_{j\in\Omega_j} \Delta(r - x_j) \tag{3}$$

where l_{jl} is (i, j) element of Laplacian L. Ω_j is connected neighbor nodes with leader state x_j. $\Delta(r - x_j)$ and ε are denoted reference error and convergence factor, respectively. We will provide Leader-Follower network model based on Eq. (2) and (3).

There are several ways of choosing leader agent; information accessibility view point and other benefit of inclusion. So, we introduce Shapley value, which is the measure for cooperation. And it is also used to calculate individual contribution for the cooperation. Otherwise, It's necessary to consider that not all agents have direct access to the leader's trajectory

C. Sharpley value

Sharpley value refers to an n-player cooperative game with transferable utility (N, v), and for each player $i \epsilon N$, the player should get the average of the expected contribution $\emptyset_i(N, v)$. N is defined as the set of all n-players, and each player i can be represented as agent in networked system. $v(\bullet)$ is the characteristic function with $v(\emptyset) = 0$. Let S be a subset of N, hence, we can obtain the characteristic function $v(S)$ under the condition $2^n \epsilon R$ and $v(R \cap C) \geq v(R) + v(C)$.
The Sharpley value can be written as follow

$$\emptyset_i(N, v) = \sum_{\substack{S \subseteq V \\ i \in S}} \frac{(s-1)!(n-s)!}{n!} (v(S) - v(S \mid \{i\})) \tag{4}$$

where $\emptyset_i(S/\{i\}, v) \equiv 0$. s is denoted the number of players in S ($s = |S|$), for each game (N, v) for each player i $(i \epsilon N \subseteq U)$.
Essentially, the Shapley value is the average expected marginal contribution of one player after all possible combinations have been considered. With the knowledge of Sharpley value, individual contribution for the cooperation can be calculated.

D. Implicit leadership reinforcement

In leaderless network system, unmatched unknown parameters will be intertwined with the Laplace matrix, making it difficult to design the updating law of distribution parameters. Hence, a concept, Implicit leadership reinforcement (ILR) is proposed by researchers, allows informed agents change their confidence so that the entire group can reach a single consensus [10]. Researchers has investigated ILR, and the results showed that all agents states will converge to the target state of the dominant group on the premise that the informed agent is connected. Yu and *et. al* proposed ILR algorithm to describe the reinforcement routine for agents [10]. For an informed agent a_i, the algorithm can be represented by a loop with the constants δ, $\bar{\rho}$, $\bar{t}$ and α.

if $\sum_{j/a_j \epsilon N_i} \frac{1\{/|x_j(t) - x_i^*|/\leq \delta\}}{/N_i/} > \bar{\rho}$ then

$$\omega_i(t+1) = \omega_i(t) + \alpha$$

else

$$\omega_i(t+1) = \max\ (\omega_i(t) - \alpha, 0)$$

if $\omega_i = 0$ (permanently) for $\bar{t}$ time steps then

a_i becomes non-informed

where $\omega_i(t)$ is a time-varying variable represents a confidence., The decision of how to change $\omega_i(t)$ is called ILR. Based on the algorithm, the performance of informed agents can be described. If the states $x_j(t)$ of a significant percentage $\bar{\rho}$ of the neighbors of informed agents are significant closed to the goal states x_i^* with a constant δ, then the confidence ω_i increases, otherwise, it decreases. Meanwhile, the agent can be non-informed if the confidence reaches 0 permanently.
In the process, any agents are able to join or leave the informed agent group or change confidence, which is flexible. At the same time, the process will continue even with updated, resulting in more effective performance.

3. GROUP CONSENSUS WITH LEADERLESS NETWORKED SYSTEM

(A) Single-input

Before discussing the leaderless network system, a dynamic leader- follower system of intelligences consisting of ten intelligences should be assumed. For the leaderless network system, the leaderless system can be approximated as a relatively dynamic intelligent body system in which the leader is randomly selected, and the system may generate one or two leaders due to one or two messages from the outside world, and each intelligent body may be the first to receive the message because it receives the message from the outside world. This is the prior condition of dynamic leader-follower.

1) single-input flocking and grouping consensus

For a single-input system, how to transfer information among the set of seven intelligences and converge the multi-intelligence system according to the single external input requires the Olfati algorithm, which was proposed by Olfati-Saber in 2006 to describe distributed swarm control algorithms in free space and in the presence of obstacle-constrained space.

In free space, using the swarm control algorithm, the control input of each mouth intelligent body contains the following three items:

$$u_i = f_i^g + f_i^d + f_i^r$$

In the above equation, the first term is a gradient-based term, the second term is a velocity-consistent parallel term that can act as a damping force, and the third term is a guidance feedback term that acts as a target term for group motion. Assuming that there are N intelligences in the n-dimensional Euclidean space, the i-th multi-intelligence dynamics equation is:

$$\begin{cases} \dot{q}_i = p_i \\ \dot{p}_i = u_i \end{cases}$$

In the above equation, q represents the position vector of multi-intelligent body i, p represents the velocity vector of multi-intelligent body i, and u represents the control input (acceleration) vector of multi-intelligent body i. At this point, the control input ui is designed with the goal of enabling control of N multi-intelligents to produce swarming behavior. Where qi , $pi, u \in R^m$, for all nodes i within $i \in v$ The position of node i is denoted by $q^i \in R^m$. The vector $q = col\,(q_1, \ldots, q_n) \in Q = R^m$ is called the configuration of all nodes in the graph composition. The interaction range of two intelligences is denoted by $r > 0$. The spherical space with radius r delineates the set of spatially adjacent multiple intelligences of intelligences i in the space, which can be defined as:

$$Ni = \{j \in v / \| q_j - q_i \| / < r\}.$$

The || · || in the above equation is the Euclidean parametrization in R^m space. Given the interaction range $r > 0$, a spatial induction graph $G(q) = (V, \varepsilon(q))$ can be defined in terms of v and the set of edges ε. If all intelligences have the same interaction range, then the network $G(q)$ becomes an undirected graph. And the behavior of the entire multi-intelligent body satisfies the three behavior rules proposed by Reynolds. If in the above multi-intelligent body kinematic equations, it is assumed that the control input $ui = ui^a$

$$u_i^a = \sum_{j \in N_i} \phi_a (\|q_j - q_i\|_\sigma)\, n_{ij} + \sum_{j \in N_i} a_{ij}(q)(p_j - p_i)$$

$$n_{ij} = \sigma_\varepsilon (q_j - q_i) = \frac{q_j - q_i}{\sqrt{1 + \varepsilon / \| q_j - q_i \|^2}}$$

In the above equation n_{ij} is the vector connecting q_i and q_j, and $0 < \varepsilon < 1$ is a fixed parameter of σ parametrization. Another algorithm can also be used to include a bootstrap feedback term, assuming that the control input is: $u_i = u_i^a + u_i$

$$u_i = \sum_{j \in N_i} \phi_a (\|q_j - q_i\|_\sigma)\, n_{ij} + \sum_{j \in N_i} a_{ij}(q)\,(p_j - p_i) + f_i^r\,(q_i, p_i)$$

$$u_i = f_i^r\,(q_i, p_i, q_r, p_r) = - c_1 (q_i - q_r) - c_2 (p_i - p_r)\,,\ c_1, c_2 > 0$$

This equation is the bootstrap feedback term in the above equation, and $(q_r, p_r) \in R^m \times R^m$ is the state vector of the γ-intelligence, which can be a dynamic or static used to represent the target of the group. Let (p_d, q_d) be a pair of fixed-valued m-dimensional vectors used to represent the starting position and velocity of the γ -intelligence, and the following dynamic equation can describe the γ -dynamic intelligences.

$$\begin{cases} \dot{q}_r = p_r \\ \dot{p}_r = f_r\,(q_r, p_r)\,, \end{cases}$$

2) Simulation and results for single-input situation

For a single-input intelligentsia consistency consensus, it is assumed that there is an external input signal detected by a random leader that converges and achieves agreement of the swarm system from a random direction in the case of information input. Using the concept of Olfati algorithm [5], the simulated situation is viewed with 10 agents and different number of iterations, for which the perceptual radius of the agents is set to 6 and all the agents are arranged in random positions and velocities, represented by a two-dimensional vector.

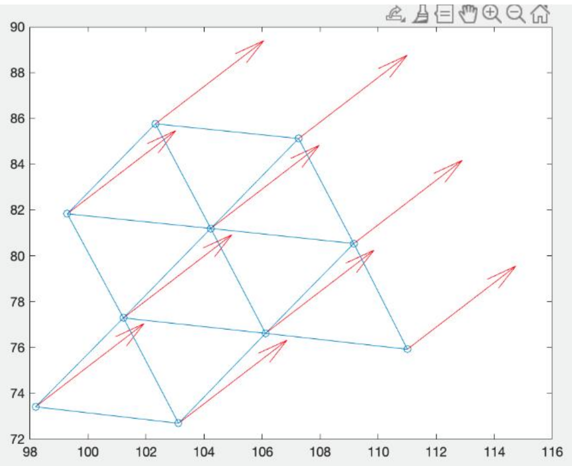

Figure: The group state of one group at t=500

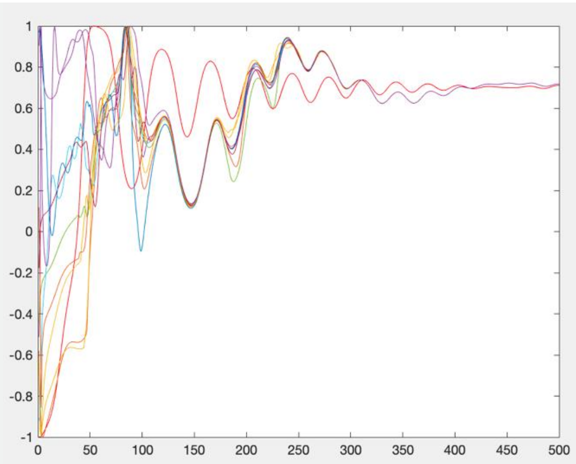

Figure: The iteration of their displacement changes

From the figure, we can get that in the initial state, the speed and position of the agents are chosen randomly, and in order to reach a consensus, the network needs time to iterate the algorithm and change the speed and the distance between the agents. When the iteration time is 500, the group consensus has been formed. It is easy to see from the simple two-dimensional position map and the convergence that the agents positions have formed a consensus, and the results of convergence after fluctuations can be seen on the iterative images of velocity and displacement, which are also converging to a point.

(B) Multi-inputs, with Shapley value.

For the multi-input, the network is considered as a multiple informed agent groups as mentioned [10]. According to the ILR algorithm, the two distinct groups are given two different goal headings which is the different goal states. However, the flocking steps to reach their consensus is the same as the single informed group. As a result, here we still use the Olfati-Saber algorithm to do the group consensus simulation. As Olfati mentioned that [5], if the algorithm does not consist of the feedback navigation term, the two grouping consensus may not be achieved, which will cause fragmentation phenomenon.

When it comes to the consensus grouping, the groups are decided intuitively based on the agent detection radius proposed by Couzin and in other related algorithm [5, 10, 12]. In conventional research, the grouping is decided by assumption. However, the agents in the middle of groups are not considered with clearly. Here, we do emphasize the decision procedure for the agent in between groups with the help of Shapley value.

Couzin presented that every grouping individual has their preference in the desired travel motion [11]. The group consists of N individuals, and each of them has a position vector $c_i(t)$, direction vector $v_i(t)$, and speed s_i, trying to keep a minimum distance α with their neighbor j. d_i represents the desired travel direction.

$$d_i(t+\Delta t) = -\sum_{j\neq i}\frac{c_j(t)-c_i(t)}{|c_j(t)-c_i(t)|} \quad (4)$$

By adjusting the distance between the neighbors, the agents could align with the Reynolds rules that the collision is avoided and the personal space is reserved. If the neighbors are not detected in the radius, the agents are tending to align with the j neighbors within the range ρ.

$$d_i(t+\Delta t) = -\sum_{j\neq i}\frac{c_j(t)-c_i(t)}{|c_j(t)-c_i(t)|} + \sum_{j=1}\frac{v_j(t)}{|v_j(t)|} \quad (5)$$

$d_i(t+\Delta t)$ is transferred into the related unit vector in the next formula.
To choose the direction of the preference, the agents are given the information from the informed group, but they are naïve initially, so the preference weighting term ω is determined by the actual social interaction in the specific situation, the gi is the preference direction of each agent, where:

$$d_i'(t+\Delta t) = \frac{\hat{d}_j(t+\Delta t)+\omega g_i}{|\hat{d}_j(t+\Delta t)+\omega g_i|} \quad (6)$$

In summary, the desired direction of the agents depends on the social interaction preference weighting, so for the simulation, it is valid to do the assumption for the grouping. In the multi-input simulation, we give two different goal state which is the final consensus states for two groups, and the agents are allocated to two groups. However, in this paper, we also consider the inbetween agent allocation. For those agents who are at the vague margin between the two groups, it is hard to decide which is the final consensus state to be reached. In this paper, the Shapley Value in the game theory is considered to decide the contribution of the inbetween agent to two groups and decided the better strategy of the grouping.

C. Illustrative example

For the basic concept of the Shapley value has been introduced in the section 2.C, it provides the calculation procedure for the cooperative grouping by calculating the marginal contribution to grouping. Simple example is introduced in Figure 3. Assume a network with 5 agents in Figure 3, a2 and a5 are assigned into group 1, while a3 and a4 are in the group 2, and the agent 1 need to decide which group it belong to.
For the marginal contribution of a1 to each group, we use equation (4) [13]. By the assumption of characteristic function of each agent and their cooperative ones are given as follow:

$$v(a1),\ldots,v(a5) = 150, v(a1,a2) = 120, v(a2,a5) = 125, v(a1,a2,a5) = 200,$$
$$v(a4,a3) = 150, v(a4,a2) = 120, v(a4,a5) = 120, v(a2,a4,a5) = 200, v(a1,a3) = 100,$$
$$v(a1,a4) = 135, v(a1,a3,a4) = 250$$

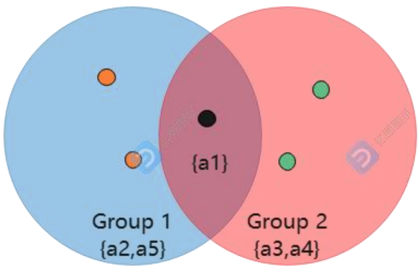

Figure 3. The example network including inbetween agent

Then, the calculation results are obtained as follows. Marginal contribution in group;

$$\text{a1 marginal contribution in group 1: } \frac{2}{6}50 + \frac{1}{6}100 + \frac{1}{6}70 + \frac{2}{6}75 = 70$$
$$\text{a1 marginal contribution in group 2: } \frac{2}{6}50 + \frac{1}{6}50 + \frac{1}{6}85 + \frac{2}{6}50 = 55.83$$

From the calculation result, agent a1 is in the group 1 as $\{a1, a2, a5\}$, the marginal contribution is larger than its inclusion in group 2. Calculation procedures are illustrated in Appendix.

In networked system, we assume multi-leaders. In this case, the agent which detect external information become leader. Here, we assume there are two external signals are detected by two random leaders, dividing the group into two sub-group with different goal state to align with. For the simulation, we have used Olfati-Saber algorithm to reach the group consensus [5], the difference between the multi-informed and the single one is the two groups do the alignment separately to reach two different goal state.

For the simulation, 10 agents are shown in Figure 4. And we divide 10 agents as two groups, then each group consists 5 of them. With the agent network, two distinct leaders get the goal state. For the agent perception radius is set to be 6, and all agents are arranged with random position and velocity, which are represent by 2 dimensional vectors.

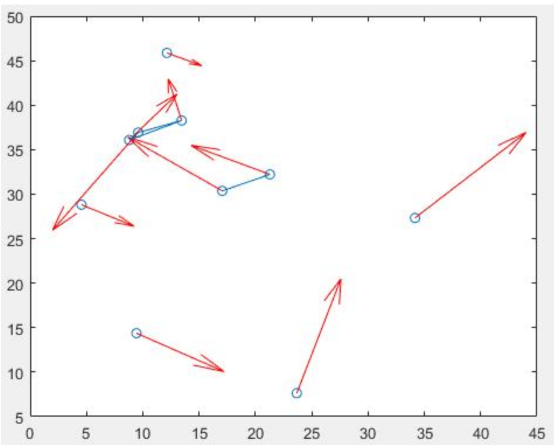

Figure 4. The initial state of the 10 agents

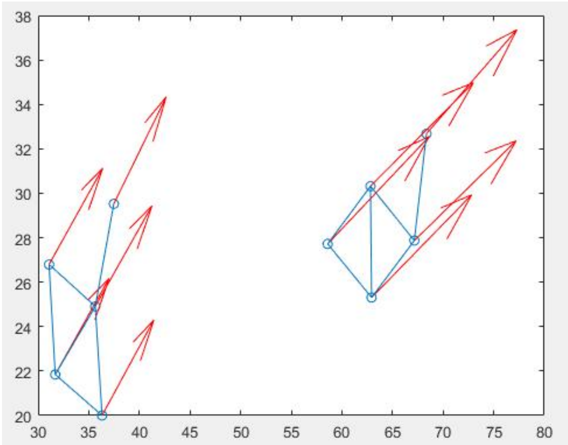

Figure 5. The group state of two sub-group at 250th step

From the simulation results, the initial state(velocity and position) of the agents are randomly selected in Figure 4, and two goal state is randomly selected for them to align with. To become consensus flocking, the network need time to iterate the algorithm to change the velocity and the distance between the agents. Figure 5 and 6 show the agent behavior after 250th step and 500th step, respectively. In Figure 7, the velocity changes along the time could be seen in a total 500-time steps range. In the situation given in this simulation, the grouping consensus had been formed at about 400th step, which could be varied from the different initial state and the goal state of the network.

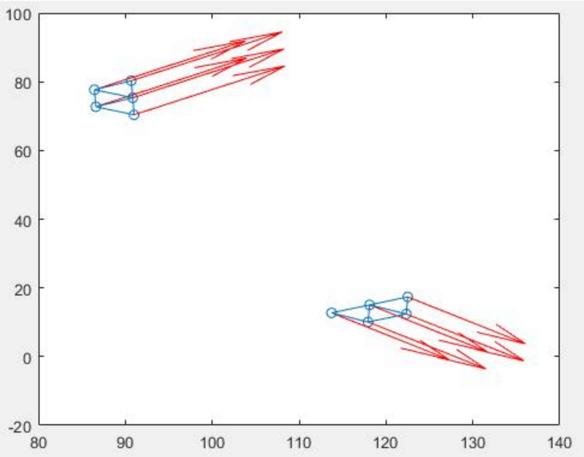

Figure 6. The group state of two sub-group at 500th step

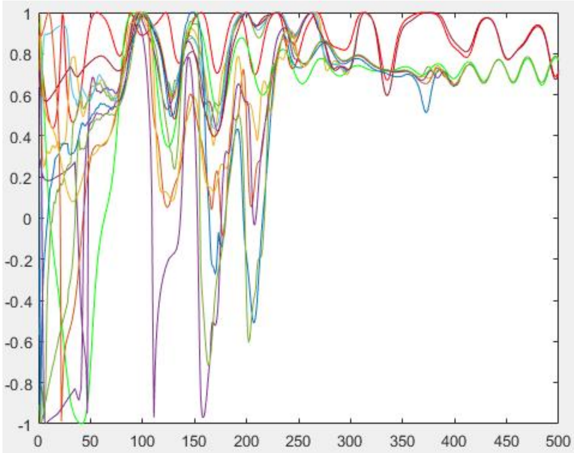

Figure 7. The iteration of their velocity changes

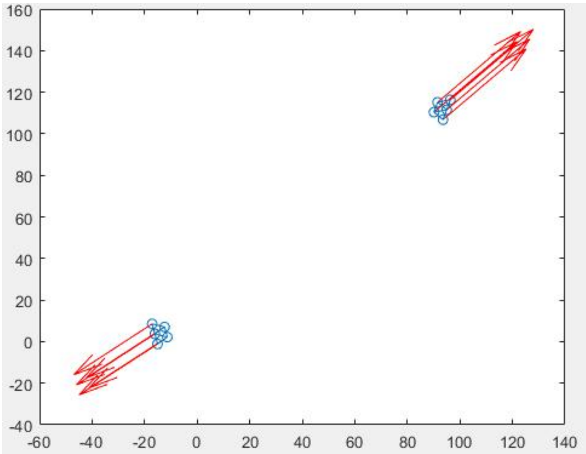

Figure 8. The group state of two sub-group at 500th step

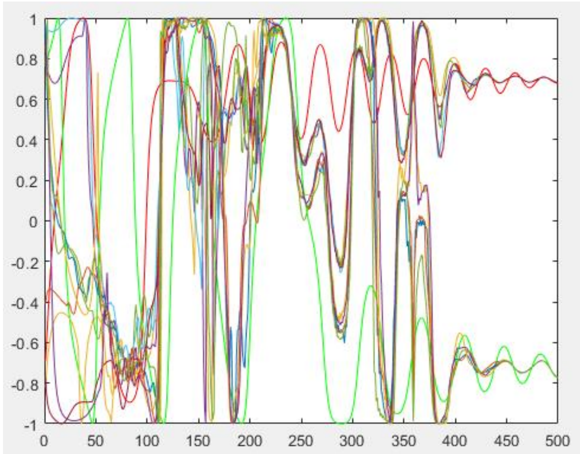

Figure 9. The iteration of their velocity changes

From the simulation results, the initial agents' states and the leaders' goal state are all randomly generated, which is to be aligned with the leaderless network reacting to uncertain external inputs.
It could be seen from the Figure 8, in another try of the simulation, the two new goal states are given as opposite direction, the two sub-group could converge to distinct final state. From the Figure 9, it could be obtained that the two groups firstly flocking together at about 90th step, and 250th step, then gradually divide different agents to their preferred direction.

4. CONCLUSIONS

In this paper, the research on the consensus in networked system has been done. We introduce the graph theory in networked system is introduced together with leader-follower structure. The difference between leader-follower and leaderless is leader choose principle. There is no specific agent being leader in leaderless networked system. In this case, arbitrary agent can be leader when an agent takes external information or commend. Furthermore, we also considered multi-information to the networked system, in this case, networked system is divided as many groups with multi-leader for each group. With the help of Shapley value formula, calculation of contribution to the group for the agent in the middle of each group has been carried by the simple example. In the simulation, we assume 10 agents in the networked system. For two external commends, two group behavior shown with different direction. Their behaviors with direction and velocity change are illustrated as well. Direction algorithm is used form the result of Olfati-Saber algorithm [5]. For the Olfati-Saber algorithm, the multi-inputs allow the external inputs larger than two, but in the simulation for this paper and other related papers, the leader number is always set to be two to show the grouping consensus. Simulation results show consensus with convergent characteristic for the networked system. However, In the process of research, we encountered some difficulties, such as there are differences among individuals in the process of multi-agent control, and secondly, the convergence process is not the fastest. The algorithms mentioned in the article can be optimized like leading feedback items, or another consensus algorithm for rapid convergence can be studied in the future.

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Appendix

Case 1: A1's marginal contribution comparison to group $\{a1, a2, a5\}$ and $\{a3, a4\}$;

$$\emptyset_i(N, v\{1\}) = \sum_{\substack{v\{1\}\subseteq V \\ i\in v\{1\}}} \frac{(1-1)!(3-1)!}{3!}(v\{1\} - v\{\phi\}) = \frac{2}{6}(50-0)$$

$$\emptyset_i(N, v\{1,2\}) = \sum_{\substack{v\{1,2\}\subseteq V \\ i\in v\{1,2\}}} \frac{(2-1)!(3-2)!}{3!}(v\{1,2\} - v\{2\}) = \frac{1}{6}(150-50)$$

$$\emptyset_i(N, v\{1,5\}) = \sum_{\substack{v\{1,3\}\subseteq V \\ i\in v\{1,3\}}} \frac{(2-1)!(3-2)!}{3!}(v\{1,3\} - v\{3\}) = \frac{1}{6}(120-50)$$

$$\emptyset_i(N, v\{1,2,5\}) = \sum_{\substack{v\{1,3\}\subseteq V \\ i\in v\{1,3\}}} \frac{(3-1)!(3-3)!}{3!}(v\{1,2,5\} - v\{2,5\}) = \frac{2}{6}(200-125)$$

Case 2: A1's marginal contribution comparison to group $\{a2, a5\}$ and $\{a1, a3, a4\}$;

$$\emptyset_i(N, v\{1\}) = \sum_{\substack{v\{1\}\subseteq V \\ i\in v\{1\}}} \frac{(1-1)!(3-1)!}{3!}(v\{1\} - v\{\phi\}) = \frac{2}{6}(50-0)$$

$$\emptyset_i(N, v\{1,3\}) = \sum_{\substack{v\{1,3\}\subseteq V \\ i\in v\{1,3\}}} \frac{(2-1)!(3-2)!}{3!}(v\{1,3\} - v\{3\}) = \frac{1}{6}(100-50)$$

$$\emptyset_i(N, v\{1,4\}) = \sum_{\substack{v\{1,4\}\subseteq V \\ i\in v\{1,4\}}} \frac{(2-1)!(3-2)!}{3!}(v\{1,4\} - v\{4\}) = \frac{1}{6}(135-50)$$

$$\emptyset_i(N, v\{1,3,4\}) = \sum_{\substack{v\{1,3,4\}\subseteq V \\ i\in v\{1,3,4\}}} \frac{(3-1)!(3-3)!}{3!}(v\{1,3,4\} - v\{3,4\}) = \frac{2}{6}(250-150)$$